

Discrete Velocity Propelled Averaged Crossover in Solving Large Scale Traveler Salesman Problem

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Abstract: The Traveling Salesman Problem (TSP) is one of the most widely studied NP-hard combinatorial optimization problems. Traditional Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA) trapped into the local minimum without reaching the optimum solution. To avoid this problem, we propose the new Discrete Velocity Propelled Averaged Crossover (DVPAC) to reinforce the diversity in the search space while keeping the optimum searching direction. This will be applied in solving used in solving practical very large scale TSP in different countries. Practical experiment shows that our DVPAC can provide very satisfactory solutions and outperforms other algorithms,

Keywords: *Traveling Salesman Problem, Particle Swarm Optimization, Simulated Annealing, Genetic Algorithm, Discrete Velocity Propelled Averaged Crossover.*

1. Introduction

The Traveling Salesman Problem (TSP) has been introduced as one of the NP – complete mathematical optimization problems in 1930s [1]. The problem is to find an optimal tour for a traveling salesman wishing to visit each of a list of n cities exactly once and then return to the home city. Such optimal tour is defined to be a tour whose total distance (cost) is minimized and has been widely studied in the fields of mathematics, graph theory, and computer science due to its theoretical and practical values [2-3].

The standard artificial intelligent search techniques have been widely applied to find the optimum tour of the TPS problems, such as, Simulated annealing (SA) [4], Particle Swarm Optimization (PSO) [5], and Genetic Algorithms (GA) [6].

Recent researches have shifted focus to employing different hybridization techniques to solve all kinds of complex large scale optimization problems like the TSP. The essence of the hybridization process is mainly to utilize the complimentary advantages and value-added information found in several algorithms and insufficient in single algorithm based approaches. For example, a hybrid model of Adaptive PSO and GA [7], another one of PSO and SA [8], also, combination of PSO, GA, and SA have been introduced showing significant improvement over traditional models [9]. However, few studies have been introduced for the proper constitution in these hybrid models, for example, the effect of the mutation operator [10] and generating offspring [11] in GA have been introduced. To do so and based on our previous researches

in stock market prediction [12] and wind speed prediction [13], we introduce in this paper the new hybrid model with Discrete Velocity Propelled Averaged Crossover (DVPAC). This paper is organized as follows: the problem formulation of TSP problem optimization is formulated in section II and section III reviews concepts of PSO, GA, and SA. Section IV introduce our new proposed DVPAC and in section V experimental study is presented for well-known practical TSP in different countries, and finally the conclusion and future work are presented.

2. TSP Problem Formulation

Let $G(V, E)$ be a complete undirected graph with vertices $V, |V| = n$, where n is the number of cities, and edges E with edge length C_{ij} for the cities (i, j) . Our work focus on the symmetric TSP case in which $C_{ij} = C_{ji}$ for all cities (i, j) where the objective function, is the minimization of the total distance to be traveled as follows [1-3] :

$$\text{Min } \sum_{i \in V} \sum_{j \in V} C_{ij} X_{ij} \quad (1)$$
$$X_{ij} = 0 \text{ or } 1 \text{ and } i, j \in V$$

Where X_{ij} is a binary constraint, $X_{ij} = 1$ if $E(i, j)$ is in the solution, $X_{ij} = 0$ otherwise to define a regular assignment problem. To ensure that each city is entered from only one other city and each city is only departed to another one, the following conditions should be satisfied.

$$\sum_{i \neq j} X_{ij} = 1 \quad (2)$$

$$\sum_{j \neq i} X_{ij} = 1 \quad (3)$$

Any sub tours S within the main tour is eliminated

$$\sum_{i \in S} \sum_{j \in S} X_{ij} \leq |S| - 1 \quad \forall S \subset V, S \neq \emptyset \quad (4)$$

3. Applying the Simulated Annealing to Particle Swarm Optimization and Genetic Algorithm in Solving TSP Problem

The common issues among all optimization problems are to avoid trapping into local minimum, increase the diversity, and keeping reinforcement direction in the search space [12-13]. Thus a hybrid model of SA, PSO, and GA is used in TSP as follows

A – Simulated Annealing (SA)

It was first introduced in solving TSP in [14] where the i^{th} route of n -city order C_i is a possible solution where $C_i = \{S_1, S_2, \dots, S_n\}$ and $S_l = k$ means that city k is visited in order l and the associated cost function $F(C_i)$ is the route C_i length. Cost function change $\Delta F(C_i)$ is the difference between the old route length C_i^{old} and new one C_i^{new} where $\Delta F(C_i) = F(C_i^{new}) - F(C_i^{old})$. Starting from an initial solution, any route C_i reduces the cost function $F(C_i)$ will be accepted, otherwise its acceptance would be according to acceptance probability P as follows [8]

$$P = \begin{cases} 1 & \Delta F \leq 0 \\ \exp\left(-\frac{\Delta F}{T}\right) & \Delta F > 0 \end{cases} \quad (5)$$

Where T is the temperature control parameter which decreases during each iteration affecting the acceptance of new routes. As the temperature decreases, the acceptance probability P of a degraded route decreases. The temperature control parameter is reduced according to cooling equation as follows

$$T_i = \alpha T_0 + T_\theta \quad (6)$$

where α is cooling coefficient arbitrary selected constant in the range between 0 and 1, T_0 is the initial temperature and T_θ is the lowest temperature value.

B – Particle Swarm Optimization (PSO)

PSO is a population based stochastic optimization technique where each particle in the swarm represents a potential solution of the optimization problem in the search space [15], but it was suitable for continuous optimization problems and could not be applied directly to discrete optimization problems like TSP, thus these

modifications have been proposed to be suited for TSP [8] as follow:

Step1: configure swarm of size m containing routes C_1, C_2, \dots, C_m where each route represents a particle in this swarm.

Step2: two cities S_u and S_v are selected randomly in route C_i and their visiting order u and v are exchanged, while the visiting order of the other cities remains the same. After exchanging, a new route C'_i is generated. Assume the city visiting order sequence of the old route is: $C_i = \{S_{u-1}, S_u, \dots, S_{v-1}, S_v\}$. After exchanging the visiting order of city u and city v , the new route is: $C'_i = \{S_{u-1}, S_v, \dots, S_{v-1}, S_u\}$

Step 3: the selected route among C_i and C'_i will be done according to the SA accepting rule equations (5-6)

Step 4: the particle best route C_{ibest} and the global best route C_{best} are adjusted according to the following equation

$$C_{ibest} = \begin{cases} C_{ibest} & F(C_{ibest}) \geq F(C'_i) \\ C_{best} & F(C_{best}) \geq F(C'_i) \end{cases} \quad (7)$$

and $C_{best} = \min(C_{ibest}) \quad \forall i = 1, 2, \dots, m$.

C – Genetic Algorithm (GA)

GA is an optimization techniques based on natural selection mechanism and crossover and mutation genetic operators, to find optimum solution and can be briefly summarized as follows [16]

Step 1: (Generation) randomly generate the swarm of m particles.

Step 2: (Evaluation) evaluate the cost function $F(C_i)$ of all swarm particles C_i .

Step 3: (selection): select the particles with the worst performance to be replaced with other better performance particles selected randomly as parents' particles from the remaining part of the swarm.

Step 4: (crossover): construct the new child particles by crossing over between selected parents' particles.

Step 5: (mutation): select randomly with equal probability one of the new children particles to be mutated.

Step 6: once the required cost function is met, terminates process; otherwise go back to Step 2 to start a new generation.

4. Discrete Velocity Propelled Averaged Crossover (VPAC)

The proposed hybrid algorithm is based on combining the idea of the standard velocity and position update rules of PSO with the ideas of selection, crossover and mutation from GA by using the crossover operator, which disperses the population preventing premature convergence by generating new particles accelerated away from their old direction [12-13, 17-18]. Thus in Discrete Velocity

Propelled Averaged Crossover (VPAC), the swarm will be divided into two portions, the first is the discarded portion ($m^*\Psi$) containing worst particles cost function where Ψ is the arbitrary selected breeding ratio and the other is the breeding portion which is the remaining ($m^*(1-\Psi)$) particles. For all routes in the discarded portion, this algorithm is applied

First: select route C_l from the discarded portion and two other routes C_j and C_k from the breeding portion.

Second: The selected breeding portion routes will be subjected to inverse mutation [7] yielding a new route C_j^{new} and C_k^{new} in new searching direction away from the direction of the discarded route C_l . Suppose that C_j is a tour of ten cities in the following sequence (6 1 5 9 7 2 3 4 8 10) and the selected starting city is 1 and selected mutation number is 6, thus the new route C_j^{new} will be (2 7 9 5 1 6 3 4 8 10).

Third: calculated the new cost functions $F(C_j^{new})$ and $F(C_k^{new})$ associated with the new generated route.

Fourth: the discarded route will be replaced by either one of the two generated routes as follows

$$C_l = \begin{cases} C_j^{new} & F(C_j^{new}) < F(C_k^{new}) \\ C_k^{new} & F(C_j^{new}) \geq F(C_k^{new}) \end{cases} \quad (8)$$

Fifth: all routes in the breeding portion of the swarm will be used only once while generating new routes replacing those of the discarded portion to ensure the diversity in the searching space.

5. Experimental Study

As The TSP problem remains NP-hard even for the case when the cities are in the planer Symmetric Euclidean problem for which the vertices are points $d_i = (q_i, p_i)$ in

the plane, and $x_{ij} = \sqrt{(q_i - q_j)^2 + (p_i - p_j)^2}$ is the Euclidean distance [1-3]. To validate the feasibility and effectiveness of our proposed DVPAC approach, we have applied it on some large scale TSP practical instances taken from [19] which are challenge problems consisting of cities in many countries in Argentina through Zimbabwe. We will compare the results of our proposed DVPAC with the results stated in this site, also the percentage of changes in cities locations in the best tour associated with best solution are stated to highlight the influence of our proposed algorithm. The selected values of the parameters used in our experiment are stated in Table 1.

Table 1: parameters values

parameter	value
initial temperature T_0	50
lowest temperature T_θ	0
cooling coefficient α	0.05
swarm size m	400

breeding ratio Ψ	0.5
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Details of selected benchmark TSP countries are summarized in Table 2.

Table 2 selected real-world TSP countries

Country	TSP name	No of cities	Optimum distance
Egypt	Eg7146	7146	171991.3444
Sweden	Sw24978	24978	852289.1738
Greece	Gr9882	9882	299347.6441
Italy	It16862	16862	555947.0702
Argentina	Ar9152	9152	837517.1353
Japan	Ja9847	9847	491320.9584
Finland	Fi10639	10639	519747.7941
Morocco	Mo14185	14185	425671.4106
Ireland	Ir8246	8246	206149.1912
Burma	Bm33708	33708	954022.8911
Vietnam	Vm22775	22775	564378.3819
Yemen	Ye7663	7663	237978.1624
Tanzania	Tz6117	6117	394278.7464
Kazakhstan	kz9976	9976	1061874.1063

The optimum distances obtained using our DVPAC compared with those stated in [19] and the number of changed cities in the optimum path is stated in Table 3. The sequence of our new obtained tour path for all these countries are published in our website of the Institute of Statistical Studies and Research, Cairo University [20].

Table 3 optimum distance and optimum path comparison

Country	Old Optimum distance	New Optimum distance	Tour difference	Percentage
Egypt	171991.3444	171972.8622	116	1.62%
Sweden	852289.1738	852274.6436	291	0.88%
Greece	299347.6441	299342.5352	116	1.17%
Italy	555947.0702	555932.3257	134	0.79%
Argentina	837517.1353	837513.5696	24	0.26%
Japan	491320.9584	491308.7967	78	0.79%
Finland	519747.7941	519742.8179	52	0.49%
Morocco	425671.4106	425660.5917	139	0.98%
Ireland	206149.1912	206139.5467	42	0.50%
Burma	954022.8911	954006.9841	357	1.06%
Vietnam	564378.3819	564370.1495	351	1.54%
Yemen	237978.1624	237964.5169	74	0.96%
Tanzania	394278.7464	394276.8462	25	0.41%
Kazakhstan	1061874.1063	1061871.1074	24	0.24%

These results clearly indicate that our proposed DVPAC algorithm gives significantly shorter distances showing that it is more reliable and more effective means to solve TSP very large scale problem. The percentage of the best tour difference is above 1% for Egypt, Burma, Vietnam, and Greece while it is less than 1% in the other countries. It is worth mentioning that our obtained results are also the best value that has been obtained so far we know. The following figures show the tour path using our proposed

DVPAC algorithm in some of these countries, refer to [20] for the whole set of figures of the optimum tour path.

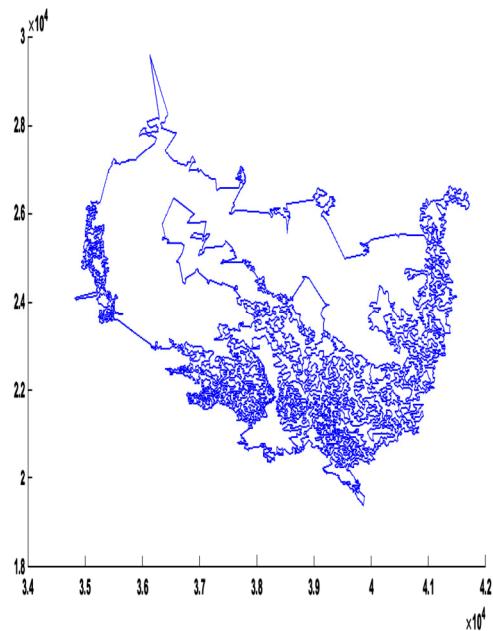


Fig.1 Tour path of Greece

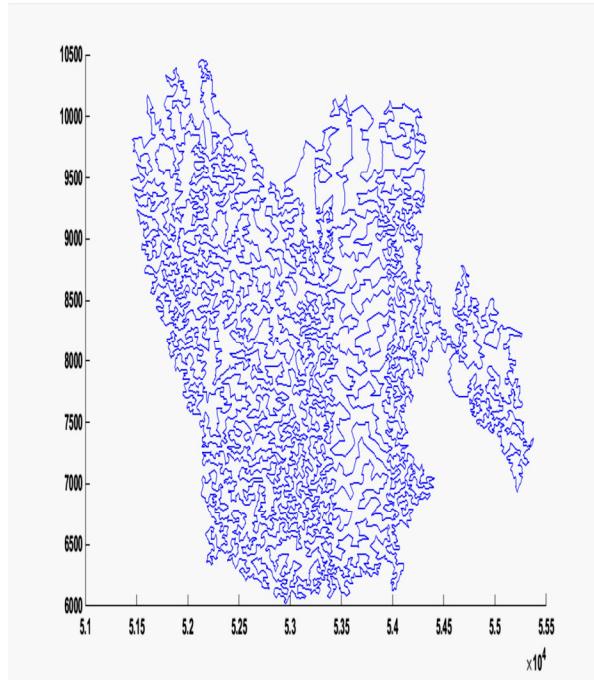


Fig.2 Tour path of Ireland

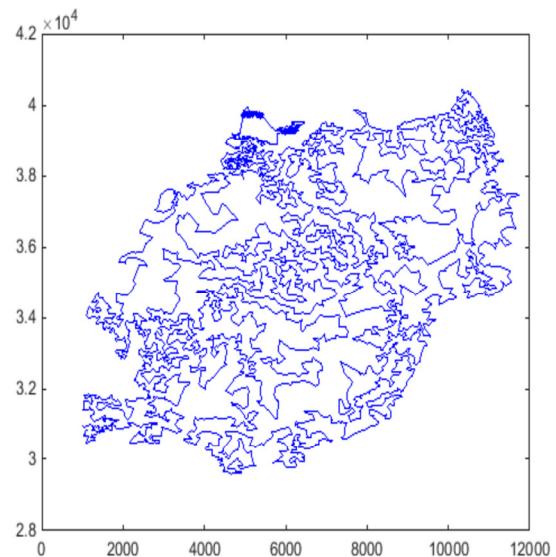


Fig.3 Tour path of Tanzania

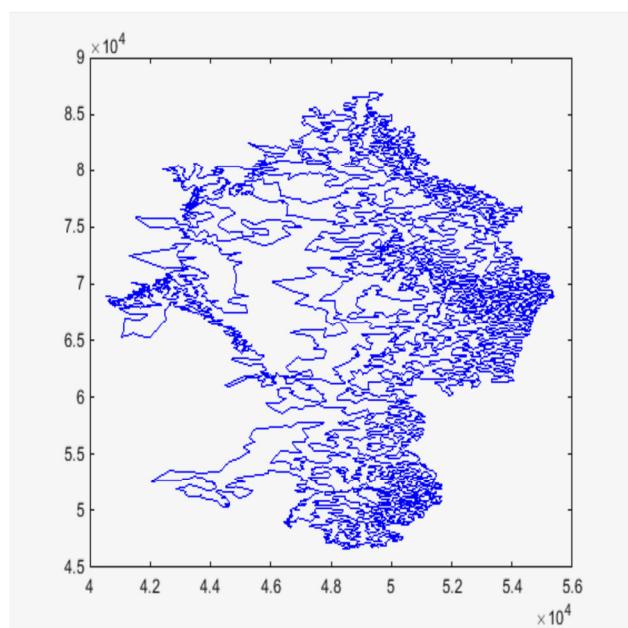


Fig.4 Tour path of Kazakhstan

6. Conclusion and Future Work

In this paper, we proposed a new hybrid algorithm based on particle swarm optimization, genetic algorithm, and simulated annealing to solve large scale TSP. This algorithm is inspired by Discrete Velocity Propelled Averaged Crossover (DVPAC) depending on dividing the

swarm into discarded and breeding portions. Well known benchmarking countries have been utilized for proving its efficiency in dealing with large scale TSP. In future work, this algorithm will be extended from the standard TSP to deal with practical cases of large number of cities like Railway Traveler Salesman Problem (RTSP). Also, it could be applied to other large scale practical real life problems like fuel consumption and petrol delivery system.

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