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Image Denoising Using A Combined System Of Shearlets and Principle Component Analysis Algorithm

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Abstract - Although image denoising is widely studied, the effect of noise in image endures in image processing. Most of the existing research works have used the basic noise reduction through image blurring but without much impact. As researchers continue to focus on the subject, it is important to appreciate the need for effective denoising methods for quality images. While some methods have managed to denoise some types of noise, in the process they affect the image quality. This research intended to establish an approach for denoising images while maintaining the image quality. To create this approach, several denoising approaches and algorithms have been studied to determine their shortcomings and a combination of two, i.e. Shearlet Transform and PCA (Principle Component Analysis Algorithm was deemed viable in adding value to the existing denoising methods. The combination method increases the superiority of the observed image, subjectively and objectively.

***Keywords* - Denoising, Image, Blur Images, Psnr,**

1. Introduction

Image denoising is an important issue in image processing but most image denoising models incorporate parameters which are closely related to the noise level. Since in often cases the noise level is unknown, the problem of choosing parameters occasionally becomes difficult, and consequently, the resulting algorithm may produce unsatisfactory images (Singh et al, 2012). During the imaging, it is inevitable for content of image to be contaminated by noise. In order to exactly extract image features or make correct analysis of the image, the reprocessing of denoising is essential. Indeed, the problem of image denoising has been studied for some decades, but there still exists gaps occasioned by the inherent complication of the inverse problem. For the existing methods, such as spatial filtering (local or non-local (Woods &Conzolez, 2008), PDE-based anisotropy diffusion (Zhang et al, 2012) transform- based threshold shrinkage (Donoho, 1995), sparse decomposition and restoration (Elad, 2006), they all have weakness and strong points. Among them, NLM and BM3D represent the state-of-the-art (Dabov et al 2007), but they suffer from high complexity and heavy computation.

2. PCA (Principle Component Analysis) Algorithm

Principal component analysis is considered as one of the most common multivariate data and signal analysis methods (Gruber et al., 2004). It transfers the correlated linearly data to un-correlated data in special domain, known also by feature space and it has many applications such as dimensional reduction in Gaussian signals and it is used in a whitening process of noisy images as well (Hyvarinen et al. , 2011. PCA can be accomplished by using eigen value corrosion of the data that contain covariance matrix. The data that has the largest eigen values may have the main data details. We use this feature to separate the pure signal from the noisy components and it gives effective results in the denoising algorithm. Image denoising by principal component analysis with local pixel grouping (LPG-PCA) was developed by (Lei et. in 2010) It is based on the assumption that the energy of a signal will concentrate on a small subset of the PCA transformed dataset, while the energy of noise will evenly spread over the whole dataset. Assume original image is denoted by $I$and noise is denoted by$v$, then the measured image will be $I_v = I + v$……..1

In order to denoiseI_v, first a train dataset $X_v$must be constructed using local pixel group. Using this $X_v$and apply PCA the noise in the image can be reduced**.**

2.1 Construct Local Pixel Group

For each pixel pxin the image, select a $K \times K$ window centered at pxdenoted by

$$x = [x_1 ... xm]^{\mathrm{T}}, m = K^2 \text{......2}$$

And a training window centered at px. The training window is $(L-K+1)^2 L \times L$
$L \times L, L > K$ …3

Take the pixels in each possible $K \times K$ block within the training block yields samples $\vec{x_i}^{v}$. If the distance between a sample and the center window $\vec{x_0}^{v}$ is smaller than some threshold, then accept the sample. So the train dataset X_v is acquired by putting all the accepted sample together as column vectors into a matrix.

2.2Denoising Using Local Pixel Group

First step of this part is centralize X_v and $\overline{X_v}$ is obtained. By computing the covariance matrix of X_v denoted by $\Omega_{\overline{x}}$, the PCA transformation matrix $P_{\overline{x}}$ can be obtained. Apply $P_{\overline{x}}$ to X_v we have

$$Y_v = P_{\overline{x}} X_v \text{....................4}$$

The covariance matrix of Y_v can also be calculated by

$$\Omega_{\overline{y_v}} = \frac{1}{n} \overline{Y_v}\, \overline{Y_v}^{\mathrm{T}} \text{..................5}$$

Shrink the coefficient of $\Omega_{\overline{y}v}$ by

$$\vec{\hat{Y}}_k = w_k \vec{\overline{Y}}_v^{k}$$

$$w_k = \frac{\Omega_{\overline{y}}(k,k)}{\Omega_{\overline{y}}(k,k) + \Omega_{v_y}(k,k)}$$

$$\text{...6}$$

And transform back to $\hat{\overline{X}}$, the noise in that pixel is reduced. Apply this to all the pixels in the image and the denoised image can be obtained. Experiments by Lei show that LGP-PCA can effectively preserve the image fine structures while smoothing noise.

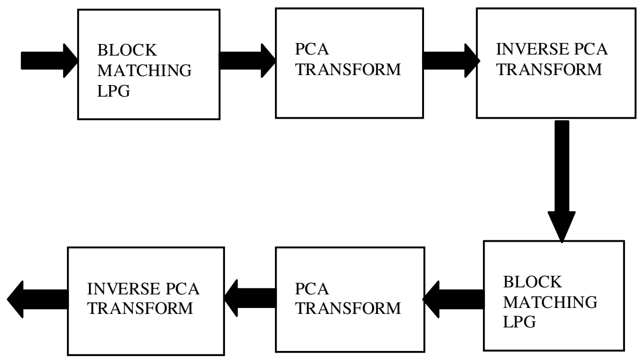

Figure 1: Stage implementation of LPG-PCA Algorithm

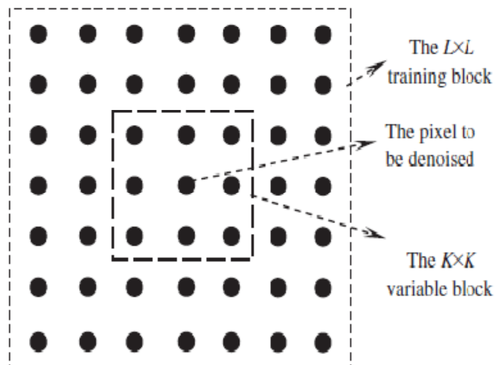

Fig. 2. Illustration of the modeling of LPG-PCA based denoising.

2. 3 Advantages and Disadvantages Of Principal Component Analysis

Advantage

Preserves local image structure during denoising

Disadvantage

PSNR is low.

3. Shearlets

Shearlets, in particular, offer a unique combination of very remarkable features: they have a simple and well understood mathematical structure derived from the theory of affine systems .they provide optimally sparse representations, in a precise sense, for a large class of images and other multidimensional data where wavelets are suboptimal and the directionality is controlled by shear matrices rather than rotations. This last property, in particular, enables a uniped framework for continuum and discrete setting since shear transformations preserve the rectan-gular lattice and is an advantage in deriving faithful digital implementations. The shearlet decomposition has been successfully employed in many problems from applied mathematics and signal processing, including decomposition of operators, inverse problems edge detection, image separation and image restoration. However, one major bottleneck to the wider applicability of the shearlet transform is that current discrete implementations tend to be very time consuming making its use impractical for large data sets and for real-time applications. For instance, the current (CPU-based) MATLAB implementation of the 2D shearlet transform, run on a typical desktop PC, takes about two minutes to denoise a noisy image of size 512 x512. The running time of the current (CPU-based) MATLAB implementation of the 3D shearlet transform for denoising a video sequence of size 192 is about five minutes.

Shearlets were introduced with express intent to provide a highly efficient representation of images with edges. In fact, the elements of the shearlet representation form a collection of well-localized wave forms, ranging at various locations, scales and orientations, and with highly anisotropic shapes. This makes the shearlet representation particularly well adapted at representing the edges and the other nisotropic objects which are the dominant features in typical images. As will be described below, these properties have direct and important implications for the efficient encoding and processing of discrete data. This is demonstrated by an increasing number of very competitive numerical applications of the shearlet transform to the analysis and processing of images and other multi-dimensional data. The significance of sparsityfor data restoration is well understood and has been addressed in seminal papers such as [20, 27]. Indeed, consider the classical problem of recovering a function f∈L2 (R) from noisy data *y*, that is, of recovering *f* from the observation $y = f + n$, where *n* is Gaussian white noise with standard deviation σ.

Finding optimal representations of signals in higher dimensions is currently the subject of intensive research. An important motivation is to obtain directional representations which capture directional features like orientations of curves in images while providing sparse decompositions. Since wavelets, although proving to be a satisfactory tool in one dimension, do not provide any directional information, several new representation systems were proposed in the past, including ridgelets and curvelets. The shearlets are an affine system with a single generating mother shearlet function parameterized by a scaling, shear, and translation parameter - the shear parameter capturing the direction of singularities. The continuous shearlet transform precisely detects the direction of singularities, in the sense of resolving the wave front set of distributions. This transform can even be regarded as matrix coefficients from a group representation of a special non-abelian group, the shearlet group, thereby providing an extensive mathematical framework for its theory, i.e., for studying the uncertainty principle related to the shearlet group aiming to derive mother shearlet functions which ensure optimal accuracy of the parameters of the associated transform. The associated discrete shearlet transform can be shown to be provide optimally sparse representations for 2-D functions that are smooth away from discontinuities along curves. Another benefit of this approach is that, again thanks to their mathematical structure, these systems provide a Multiresolution analysis similar to the one associated with classical wavelets, which is very useful for the development of fast algorithmic implementations.

3.1 Some Approaches of Shearlets

3.1.1 Discrete Shearlet Transform (DST)

Discrete shearlet transform (DST) provides efficient multiscale directional representation. The implementation of the transform is built in the discrete framework based on a multiresolution analysis. Sharp image transitions or singularities such as edges are expensive to represent and integrating the geometric regularity in the image representation is a key challenge to improve state of the art applications to image compression and denoising. To exploit the anisotropic regularity of a surface along edges, the basis must include elongated functions that are nearly parallel to the edges. Several image representations have been proposed to capture geometric image regularity. They include curvelets contourlets and bandelets. In particular, the construction of curvelets is not built directly in the discrete domain and they do not provide a multiresolution representation of the geometry. In consequence, the implementation and the mathematical analysis are more involved and less efficient. Contour lets are bases constructed with elongated basis functions using a combination of a multi-scale and a directional filter bank. However, contour let shave less clear directional features than curvelets, which leads to artifacts in denoising and compression. Bandelets are bases adapted to the function that is represented. Asymptotically, the resulting bandelets are regular functions with compact support, which is not the case for contour lets. However, in order to find bases adapted to an image, the bandelet transform searches for the optimal geometry. For an image of N pixels, the complexity of this best bandelet basis algorithm is O (N3=2) which requires extensive computation .Recently, a new representation scheme has been introduced . These so called shearlets are frame elements which yield (nearly) optimally sparse representations. This new representation system is based on a simple and rigorous mathematical framework which not only pro-vides a more flexible theoretical tool for the geometric representation of multidimensional data, but is also more natural for implementations. As a result, the shearlet approach can be associated to a multiresolution analysis. However constructions proposed do not provide compactly supported shearlets and this property is essentially needed especially in image processing applications. In fact, in order to capture local singularities in images efficiently, basis functions need to be well localized in the spatial domain.

3.1.2 Continous Shearlets

Continuous wavelet transform has the ability to identify the set of singularities of a function or distribution *f*. It was recently shown that certain multidimensional generalizations of the wavelet transform are useful to capture additional information about the geometry of the singularities of *f* . Consider the continuous shearlet transform, which is the mapping f∈$L^2(R^2)$→SHψf(a,s,t)= ⟨f,ψ*ast*⟩ f∈L2(R2)→SHψf(a,s, t)= ⟨f,ψast⟩ , where the analyzing elements ψ*ast*ψast form an affine system of well localized functions at continuous scales a>0a>0, locations t∈R^2t∈R2, and oriented along lines of slope s∈Rs∈R in the frequency domain. We show that the continuous shearlet transform allows one to exactly identify the location and orientation of the edges of planar objects. In particular, if $f = \sum_{n=1}^{N} f_n \chi_{\Omega_n}$ f=∑n=1Nfnχ Ωn where the functions fnfn are smooth and the sets ΩnΩn have smooth boundaries, then one can use the asymptotic decay of SHψf(a,s,t)SHψf(a,s,t), s a→0a→0 (fine scales), to exactly characterize the location and orientation of the boundaries ∂Ωn∂Ωn. This improves similar results recently obtained in the literature and provides the theoretical background for the development of improved algorithms for edge detection and analysis.

An example of a continuous shearlet can be constructed as follows: Let ψ1be a continuous wavelet with^ψ1∈C∞(R) and supp^ψ1⊆[−2,−12]∪[12,2], and letψ2 be such that ^ψ2∈C∞(Rn−1) and supp^ψ2⊆[−1,1]n−1. Then the functionψ∈L2(Rn) defined by^ψ(ω) =^ψ(ω1,ω) =^ψ1(ω1)^ψ21ω1ωis a continuous shearlet. The support of^ψisdepiced for ω1≥0

3.2 Merits and Demerits of Shearlets

Merits:

1 Image enhancement
2 Image separation
3 Edge detection -the continuous shearlet transform is able to precisely capture the geometry of edges

Demerits:

1. It is a complex method
2. It causes unwanted non-smooth artifact

4. Combination of Shearlet and Principle Component Analysis

This proposed method is by combining shearlet denoising approach and PCA denoising approach.

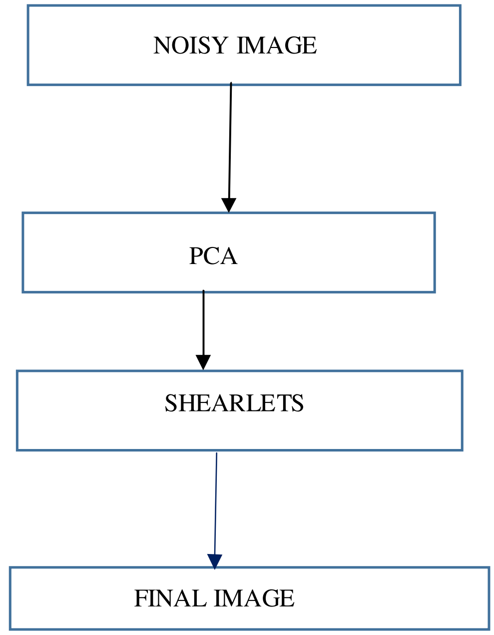

Fig 3: Combined Method Diagram

5. Results

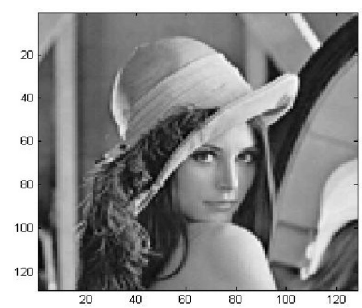

Fig 4 a Original Image

Fig 4 b Noisy image σ =30

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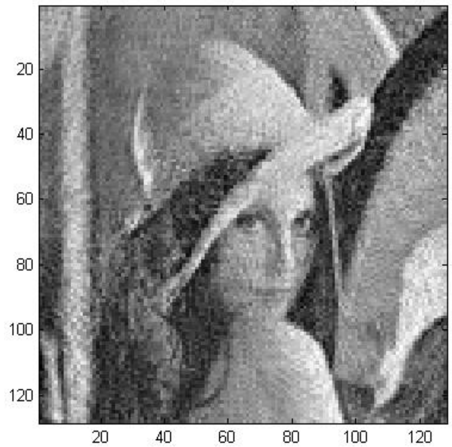


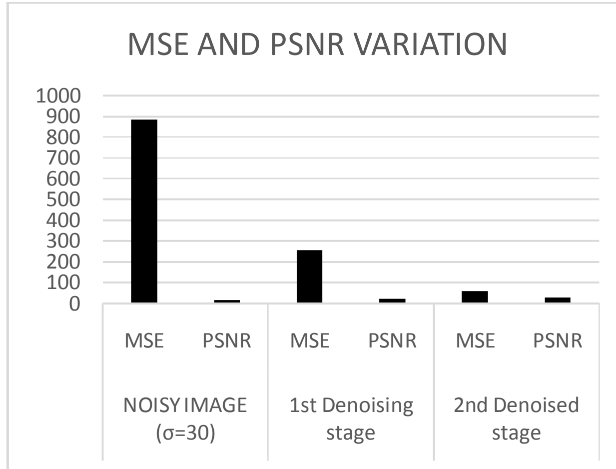
Fig 4c Denoised by PCA

Fig 4d final image denoised by shearlets

Figure 4 a is the original image which is noise free. Figure 4b is a noisy image with Gaussian noise σ=30. Figure 4 c is the image obtained after 1st denoising stage of PCA. Figure 4d is the final image obtained after second stage of denoising using shearlets.

Table 1: Values of MSE and PSNR of Lena Image after 1st& 2nd stage of denoising

NOISY IMAGE (σ=30)		1st Denoising stage		2nd Denoised stage	
MSE	PSNR	MSE	PSNR	MSE	PSNR
884.74	18.7	256.1	24.1	59.3	32.5



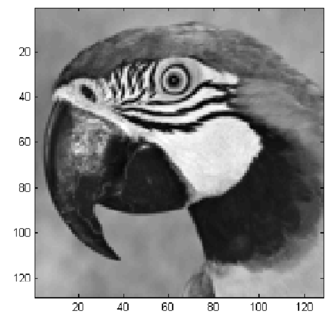


Figure 5a Original image

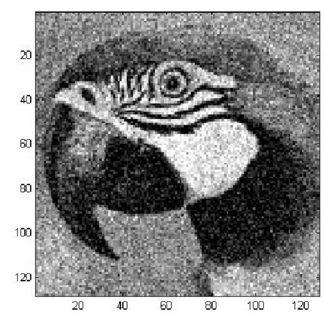


Figure 5b Noisy image

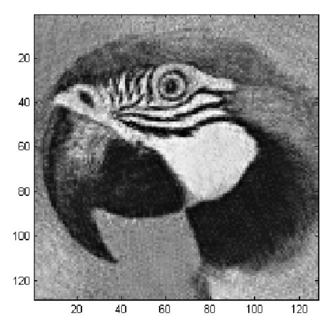


Figure 5c denoised by PCA

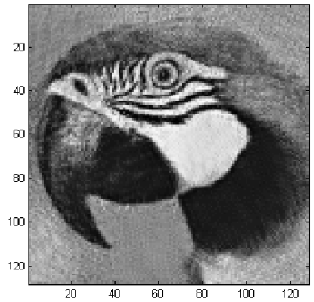
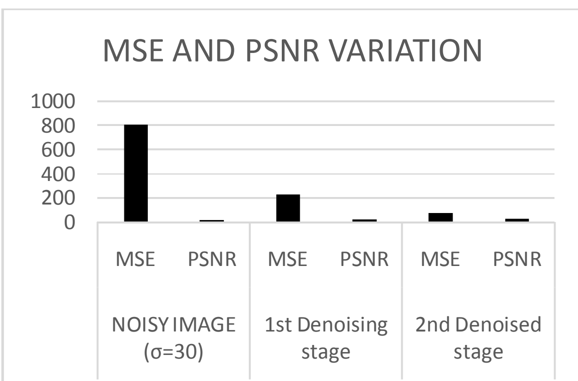


Figure 5d denoised by shearlets

Table 2: Values of MSE and PSNR of Parrot Image after 1st& 2nd stage of denoising

NOISY IMAGE (σ=30)		1st Denoising stage		2nd Denoised stage	
MSE	PSNR	MSE	PSNR	MSE	PSNR
806.6	19.1	231.2	24.51	` 81.6	29.0



6. Discussion

The quality of the image can be analyzed by both human visual and by using PSNR values. When the image of Lena with Gaussian noise σ=30 is denoised by separate methods, shearlet gives PSNR value of 30.6 and PCA gives 24.1. When the two methods are combined, the final PSNR value is 32.5 which is higher than for the separate methods. When the image of parrot with Gaussian noise σ=30 is denoised by separate methods, shearlet gives PSNR value of 26.3 and PCA gives 24.51 When the two methods are combined, the final PSNR value is 29.0 which is higher than for the separate methods

Table: 3 Review Table

Name	**comments**	**address**

7. Conclusion

The combined method gives better results both byhuman visual and by PSNR values Similarly, when the two methods are used the properties of shearlet to enhance and detect image edges and the property of PCA of preserving image structures improves the final mage.

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